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Calculation and Measurement of the Axial Forces Exerted by Photonic Tweezers

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INTRODUCTION

Although photonic tweezers are used regularly in biological research (Block, et al. 1990, Finer et al. 1994, Perkins et al. 1994), the forces they exert on transparent particles with radii larger than half the laser wavelength are still not well characterized. Commonly accepted theories are an electromagnetic model (Barton et al. 1988, Wright and Sonek 1993, Wright et al. 1994) for bead radii smaller than half the laser wavelength and a ray-optics model (Ashkin 1992, Roosen 1979, Wright et al. 1990) for beads much larger than the laser wavelength ($r \geq 5\ \lambda$). The electromagnetic model has been used to calculate radiation forces for particle radii up to 0.5 μm in water (Wright and Sonek 1993). The same theory has been applied to 2.5 μm-radius water droplets in air (Barton et al. 1988) and to 20 μm-radius micro spheres with a refractive index of 1.2 in vacuum (Visscher and Brakenhoff 1991). No such theoretical studies have been published for latex beads of that size in water. An extension of the geometrical-optics model to particle radii below 8 λ has been attempted using the properties of a Gaussian beam (Bakker-Schut et al. 1991), but the validity of this approach could not be shown. Another promising calculation for particles with a radius of 5 λ has been compared to an indirect measurement (Wright et al. 1990). In this case the trapping region was investigated experimentally but no direct force measurement has been performed to compare theoretical and experimental results. This extended geometrical-optics model (Wright et al. 1990) has not been applied to particles with radii between 1/2 λ and 5 λ.

In this contribution axial trapping forces exerted by photonic tweezers on dielectric latex micro spheres are investigated theoretically and experimentally for particles with radii between 0.25 μm and 3 μm (1/4 λ and 3 λ). Currently discussed theories have either not been validated thoroughly by experiments or fail to predict forces for particles of this size. Using the results of a theoretical approach that uses the properties of Gaussian beams and takes the size of the micro sphere and its sub-microscopic interaction with the surrounding medium into account an analytical phenomenological expression is developed that describes the axial forces as a function of the particle radius. For micro spheres with radii ≥ 1 μm the theoretical results of a sub-microscopic theory (Fig. 1) agree well with the measured forces. The phenomenological theory (Fig. 2 and 3) describes all experiments excellently. Since the axial force is smaller than the lateral force we assume that

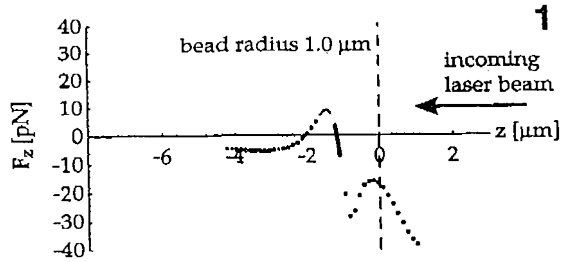

Fig. 1. Calculated axial forces for a particle (η=1.57) with a radius of 1 μm at various positions along the optical axis. The assumptions are: wavelength 1064 nm, power 160 mW, and an oil-immersion microscope objective lens (NA = 1.3) with an ideal beam waist radius of 0.43 μm. The geometric focus (z = 0) is indicated by the dashed lines. The region with positive axial forces is the trapping region. The transition from positive to negative forces at $z \approx (r + 0.2$ μm) is the equilibrium position.

experiments based on loosing particles from photonic tweezers are sensitive for the axial force.

EXPERIMENTS

The basic idea of the experiments is to capture a latex sphere of radius (r) and to move it against the forces of friction through a solution of known viscosity (η). The bead is gradually accelerated until it drops out of the photonic trap. At this maximum velocity (v_{max}) the frictional force is greater than or equal to the axial force of the photonic tweezers. This is a direct force measurement, since Stokes' law for viscous media (Landau & Lifschitz 1981, p.73)

$$F_{max}=6\pi\eta r v_{max} \qquad (1)$$

directly relates the drag force (F) and the velocity (v_{max}) of a spherical particle. It should be noted, that a particle dragged through a viscous medium has no significant macroscopic momentum. As soon as the photonic tweezers loose their influence on the particle movement is dominated by diffusion.

Theory

The phenomenological theory provides a formula that can be fitted to the data if we assume that the force is proportional to the radiation pressure and that the lateral forces cancel each other along the optical axis:

$$F_z(r) = \frac{n_{med} \cdot P}{c} \cdot q(n_p, n_{med}) \cdot \frac{\left(W_0 \cdot B \cdot Exp\left[\frac{2 r^2 z_0^2}{W_0^2 B^2}\right] \cdot \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \cdot Erf\left[\frac{\sqrt{2}\, r z_0}{W_0 B}\right] - 2 r z_0\right)}{W_0 \cdot B \cdot Exp\left[\frac{2 r^2 z_0^2}{W^2 B^2}\right] \cdot \left(\frac{\pi}{2}\right)^{\frac{1}{2}}} \qquad (2)$$

Erf is the error function, $B=(r^2+z_0^2)^{\frac{1}{2}}$, P is the power of the incoming light, c is the speed of light, and n_p and n_{med} are the refractive indices of the particle and the medium respectively. r is the variable, W_0 and z_0 are related. Thus only W_0 and q remain unknown. The parameters were the forces as a function of the bead radius. The curve was therefore fitted non-linearly to the experimental data set (Fig. 2).

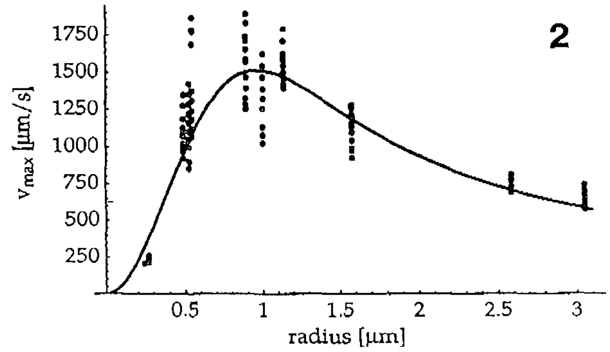

Fig. 2. Measured maximum velocities of latex spheres (η=1.57) with radii between 0.24 µm and 3.05 µm as a function of the spheres radii. The curve summarizes 237 experiments with 2 different materials and 11 different particle radii. The calculated fitting curve is obtained from Eq. 2. The fitting parameters were the lateral extent (W_0 = 0.92 µm) of the Gaussian focal point and the factor $q(n_p, n_{med})$ = 0.04.

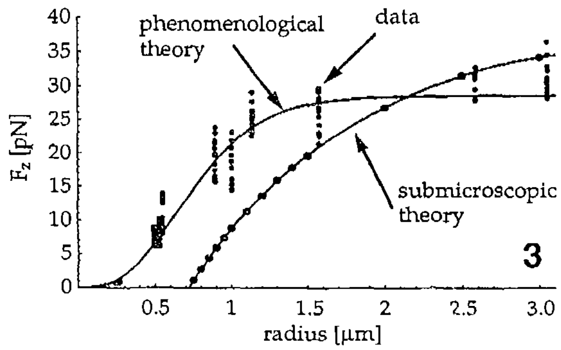

Fig. 3. Comparison of forces derived from velocity measurements as shown in Fig. 2 and calculated forces. The experimental values were obtained using Eq. 1.

DISCUSSION

The sub-microscopic theory is an explicit calculation of a scattering cross-section and thus tells us a lot about the behavior of latex beads and probably of every other transparent object caught in an photonic trap (Fig. 1). Looking at Fig. 2 which shows escape velocities as a function of the particle radius one can distinguish three different regions. Latex spheres with a size of approximately the extent of the microscope's point spread function at a wavelength of 1064 nm are captured most effectively. Above this size, beads are moved slower since the increase in radius dominates the low increase in of axial force. The velocity and thus the force for beads smaller than the extent of the point spread function rapidly approaches zero. The steep slope for small particles results from a r^3-increase of the light intensity that interacts with the volume of the micro sphere (see Eq. 2). This volume effect for small particles is typical for a scattering problem and has been observed previously (Wright and Sonek 1993).

Although in the experiment the trapped particles were moved in the x/y-plane the escape velocity is related to the axial forces and not to the lateral forces as done by some authors (e.g., Sato et al. 1991). Our interpretation is in agreement with other theoretical results, which state that lateral forces are much stronger than axial forces (Ashkin 1992, Malmqvist and Hertz 1992).

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